

Effect of a Collapsing Cluster on the CMB temperature and Power Spectrum

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We present a new model for the formation of spherically symmetric clusters in an expanding Universe. Both the Universe and the collapsing cluster are governed by the same pressure less fluid equations for which a uniform initial density profile is assumed. A simple perturbation imposed on the initial velocity field gives rise to an over-density which closely models real clusters. The computation of photon paths allows us to evaluate the gravitational effects imprinted on a Cosmic Microwave Background (CMB) photon passing through such an evolving mass. We also consider the lensing properties of collapsing clusters and investigate the effect of a population of such clusters on the primordial microwave background power spectrum.

1 Introduction

There exist several mechanisms which produces anisotropies on the CMB: the primary anisotropies which take place at the last scattering epoch and the secondary anisotropies such as the Sunyaev-Zel'dovich effect which occurs while the CMB photons are travelling through the universe. We will concentrate here on a particular secondary anisotropy first discussed by Rees & Sciama (1968). As pointed out by these authors, there is a gravitational effect on the CMB photons while they are crossing evolving cosmic structures such as a collapsing cluster of galaxies or expanding voids. Indeed photons climb out of a slightly different potential well than the one into which they entered. This effect is usually referred as the Rees-Sciama effect.

Early approaches to this problem were based on the “Swiss Cheese” model (Rees & Sciama (1968), Dyer (1976), Kaiser (1982), Nottale (1982) & Nottale (1984)), whereas more recent attempts have used the continuous Tolman-Bondi solution (Panek (1992), Sáez et al. (1993)). Our aim is to present here an improved model which, for the first time, treats such a problem exactly (see section 2).

In section 3 we apply our model to a rich galaxy cluster at a redshift $z = 0.08$. We investigate the properties of the collapsing cluster and verify that its density-profile is of a realistic shape. We then give, in section 4, an upper limit for the temperature decrement that such an evolving cosmic structure could imprint on the CMB. Our result is compared to previous works.

Section 5 deals with distant clusters ($z = 1$) since it is reasonable to assume that, at such an epoch, those objects were in a stage of formation and that our spherical collapse model would be suitable. The focus is put on the gravitational lensing phenomenon and how this would distort the observed CMB power spectrum.

We finish in section 6 by a short conclusion and discuss the applicability of our work to model the microwave decrement observed towards the, possibly lensed, quasar pair PC1643+4631 A&B.

The methods and results introduced in this poster are discussed in further details in forthcoming papers (Lasenby et al. (1997), Dabrowski et al. (1997)).

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2 Model

In this poster we are concerned with the formation of spherical galaxy clusters (i.e. pure infall) in an expanding background universe. As in the Tolman-Bondi model applied by Panek (1992), the evolution of both the collapsing cluster and the expanding Universe are governed by the same pressure-less fluid equations. Initially, the density of our fluid is uniform. Only the velocity field is perturbed in order to form a realistic cluster. In a given cosmology, this perturbation is controlled by two parameters (r_0 and a) as illustrated in figure 1. r_0 is related to the size of the perturbation and a to its rate of growth. Those parameters are free and we choose them so that the resulting observed cluster is realistic (see section 3).

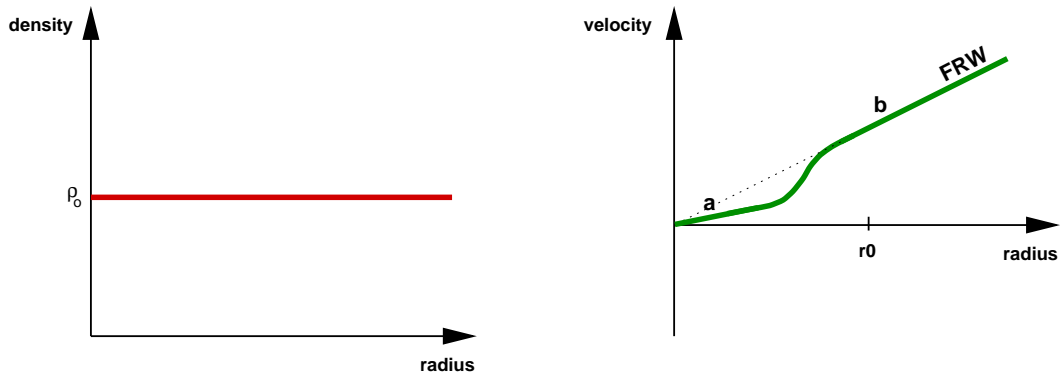


Figure 1: Fluid initial conditions for our model. Two parameters control the perturbation: r_0 and a .

We find that our model has many advantages and treats the problem much more rigorously than older attempts, particularly in the case of works based on the “Swiss Cheese” (SC) models. Indeed, as shown in figure 2, the SC models deals with discontinuous density and velocity distributions since an unrealistic vacuum region is needed to separate the perturbation from the Universe. Another improvement made with regard to the SC models is that our perturbation is defined by only two free parameters rather than three in the SC case.

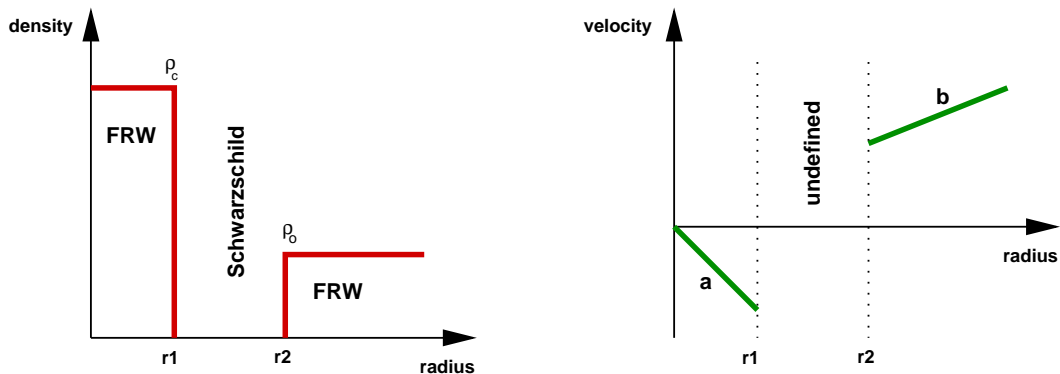


Figure 2: Fluid initial conditions for Swiss-Cheese models. Three parameters control the perturbation: ρ_c , r_1 and a .

More recent studies of the problem use the Tolman-Bondi solution (Panek (1992), Sáez et al. (1993)), which gives a similar approach to our own. In particular, both the forming cluster and the external Universe are treated as a whole and the fluid distributions are continuous. We believe our model is a refinement of the work based on the Tolman Bondi solution. Our mathematical formulation is clearer and does not need any approximation. The underlying gauge theory of gravity (Lasenby, Doran & Gull (1997)) that we use deals straightforwardly with observable quantities such

as the photon energy (Lasenby et al. (1997)). Therefore, we do not need to remove a posteriori dipole nor quadrupole-like anisotropy from the obtained CMB fluctuation (e.g. Panek (1992)).

3 Cluster properties

In this section, we model the formation of a very rich Abell cluster. The characteristics (see table 1) are chosen to be similar to those of the clusters described in Quilis et al. (1995) and Panek (1992) so that our results can be compared.

Table 1: Cluster characteristics.

	$H_o = 50$	$H_o = 100$
Redshift	0.08	0.08
Distance	450 Mpc	225 Mpc
Max density	$0.7 \times 10^4 \text{ p m}^{-3}$	$1 \times 10^4 \text{ p m}^{-3}$
Core radius	0.46 Mpc	0.23 Mpc
Mass $r < 4 \text{ Mpc}$	$1.3 \times 10^{16} \text{ M}_\odot$	$5.8 \times 10^{15} \text{ M}_\odot$

The cluster is placed at a redshift $z = 0.08$, with a core radius $R_c = 0.23 h_0^{-1} \text{ Mpc}$ (R_c is defined as the radius at which the cluster energy falls to one-half its maximum value). The maximal baryonic density ρ_{max} encounter by an observed photon is taken to be $10^4 h_0^{1/2} \text{ protons m}^{-3}$ and the baryon to dark-matter mass ratio is assumed to be 0.1. The h -dependencies for R_c and ρ_{max} ensure that the observed cluster characteristics (i.e. angular size and X-ray luminosity) are independent of the Hubble parameter. Throughout this paper we assume $\Omega_0 = 1$.

Figure 3 shows the fluid density and the fluid velocity as a function of proper time as experienced by a photon which travels straight through the centre of the cluster. From the figure we see clearly that the fluid distributions are continuous.

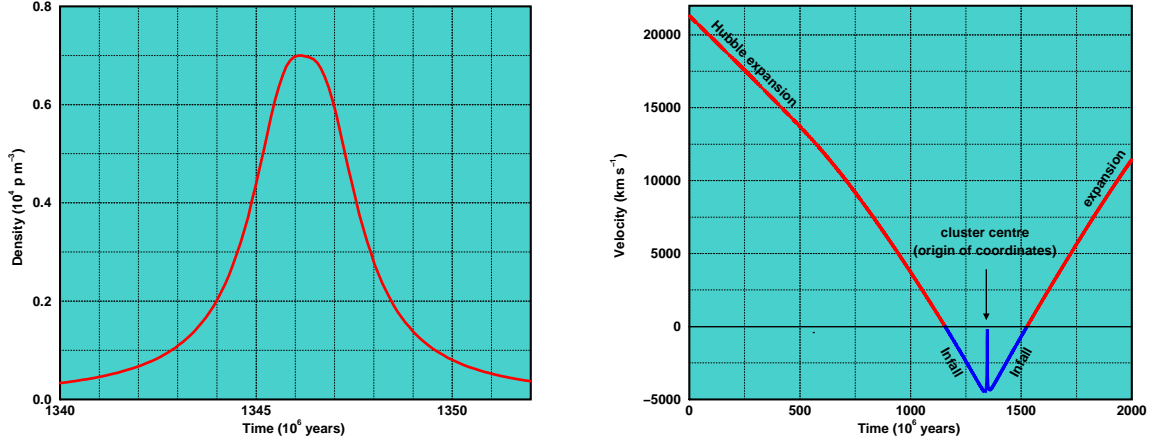


Figure 3: Plot of the fluid density (left) and fluid velocity (right) as experienced by the photon while it is travelling through the Universe and the collapsing cluster. The horizontal axis marks the time prior to the present epoch in millions of years. We assume $h_0 = 0.5$.

We next consider (fig 4) the density profile of the cluster at the time $t_c = 1436.1 \text{ Myr}$ ago, when the photon experienced the maximum density $0.7 \times 10^4 \text{ protons m}^{-3}$. In order to check that the obtained density distribution is of realistic shape, we fit our profile to the equilibrium spherical King model

(King (1966)):

$$\rho(r) = \rho_{\max} \left[1 + \left(\frac{r}{R} \right)^2 \right]^{-3\beta/2}, \quad (1)$$

where ρ_{\max} is the central density of the cluster, R represents some core radius and β is the conventional power-law index for such models. The results of a simple least-squares fit are $\rho_{\max} = 0.73 \pm 0.05 \times 10^4 \text{ protons m}^{-3}$, $R = 0.52 \pm 0.05 \text{ Mpc}$ and $\beta = 0.8 \pm 0.15$. The result of this fit is very encouraging since it shows that our model leads to a radial density profile that matches quite closely those of observed clusters.

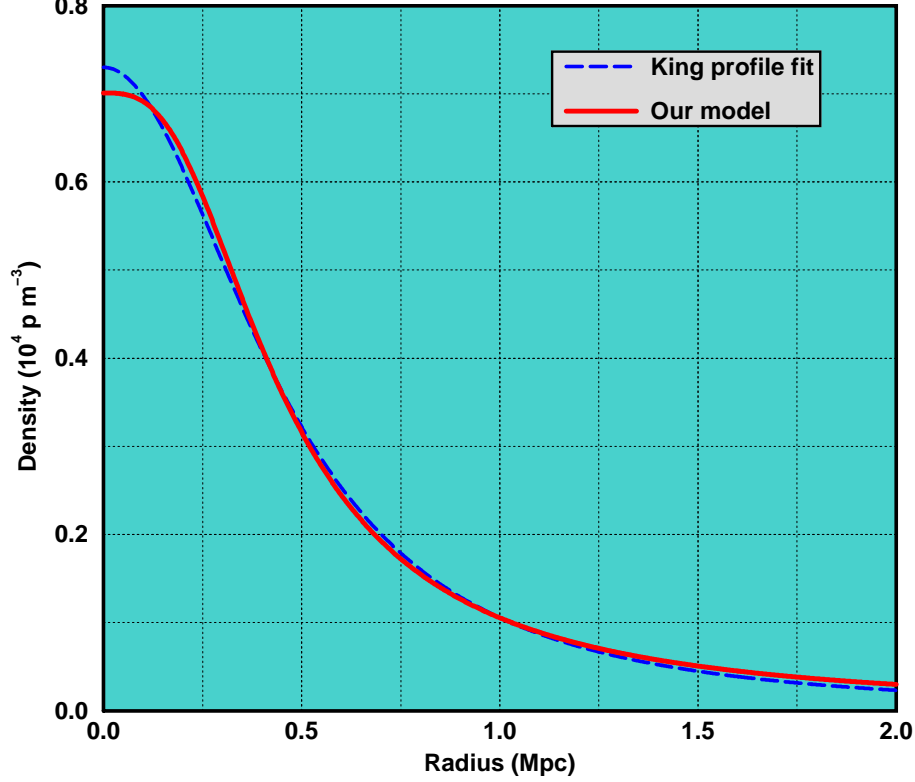


Figure 4: Plot of the density when the photon is at the centre of the cluster. The obtained profile is realistic since it can be fitted by a spherical King model.

4 Effect on the CMB temperature

We now consider photon energy and concentrate on the CMB anisotropy produced when the photon passes through the cluster described in section 3. One must remember however that our model assumes a pressure-less fluid and so the cluster, in pure-infall, might evolve too rapidly. Therefore the temperature perturbations given in this section should be considered as upper limits.

Figure 5 shows the CMB anisotropy due to the gravitational perturbation of the cluster described in table 1. The maximum temperature distortion occurs at the centre of the cluster and has the value $\Delta T/T = -0.96 \times 10^{-5}$ and $\Delta T/T = -5.2 \times 10^{-5}$ for $h_0 = 1$ and $h_0 = 0.5$ respectively. One notices that the distortion extends to rather large projected angles (e.g. $\Delta T/T \sim -1 \times 10^{-6}$ for an observed angle of ~ 2.5 degrees in the $h_0 = 0.5$ case). We may also point out the fact that the anisotropy becomes slightly positive at large angles.

We can compare the central decrements calculated above with those of previous authors. For Panek (1992) type I and type II clusters, the calculated central decrements are $\Delta T/T = -1.5 \times 10^{-6}$

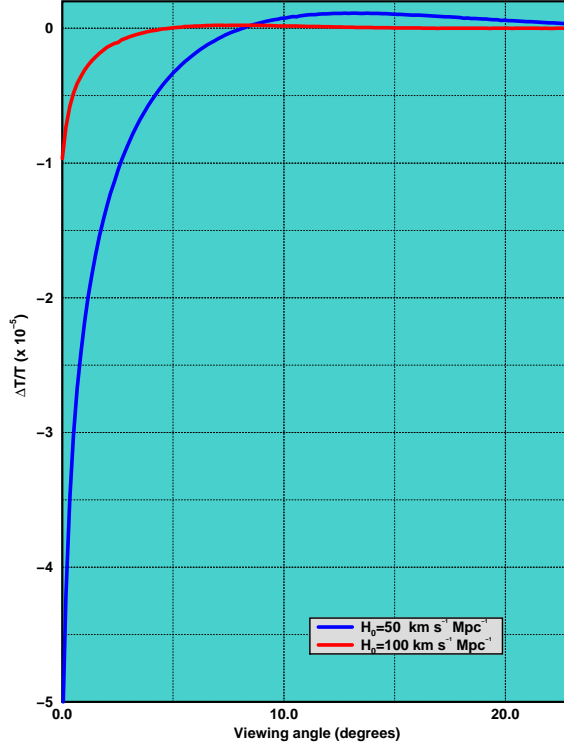


Figure 5: Temperature perturbation $\Delta T/T$ imprinted on the CMB as a function of the observed angle from the centre of the cluster.

and $\Delta T/T = -6.0 \times 10^{-6}$ respectively, whereas Quilis et al. (1995) quote the decrement $\Delta T/T = -1.2 \times 10^{-5}$. For two cluster models with similar physical properties Chodorowski (1991) finds $\Delta T/T = -7.7 \times 10^{-6}$. Finally Nottale (1984) used the SC model to predict a considerably larger central decrement of $\Delta T/T \sim 10^{-4}$. However, this last value corresponds to a very dense, unrealistic cluster. We can notice that, for $h_0 = 1$, our predicted value is in rough agreement with Panek (1992) type II, Chodorowski (1991) and Quilis et al. (1995). However our result for $h_0 = 0.5$ is about five times larger since previous works predict the same value whatever is h_0 . We then suggest that, in such a Universe, the effect on CMB photons may be more significant than previously stated.

5 Distant clusters

We consider, in this section, the formation of a massive cluster located at a redshift $z = 1$. It is more reasonable to apply our model for a distant cluster rather than the one described in section 3. Indeed, at such redshifts one could expect galaxy clusters to be in a formation process and display large inward radial velocities. As from now, the cosmology is taken to be $\Omega_0 = 1$ and $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The cluster characteristics are the following: $z = 1$, maximum density $= 1.0 \times 10^4 \text{ p m}^{-3}$, core radius $= 0.46 \text{ Mpc}$ and the mass within 4 Mpc is $1.9 \times 10^{16} M_\odot$. In the next section we investigate the lensing effect caused by such a cluster as well as the effect imprinted on the CMB temperature and power spectrum.

5.1 Gravitational lensing and effect on the CMB

A massive cluster such as described above is a powerful gravitational lens. This effect is illustrated in figure 6. Our model also allows a quantitative study of the effect of dynamically evolving lenses on the CMB fluctuations. By simulating maps of fluctuations in the CMB due to inflation and/or topological defects, we can investigate how features in these maps are affected by the presence of the lens. As an example, figure 7 shows the effect of our collapsing cluster on a CMB fluctuations realisation. The result is a central decrease of temperature at the centre (i.e. $180 \mu\text{K}$ in our case) as well as a global

magnification of the features.

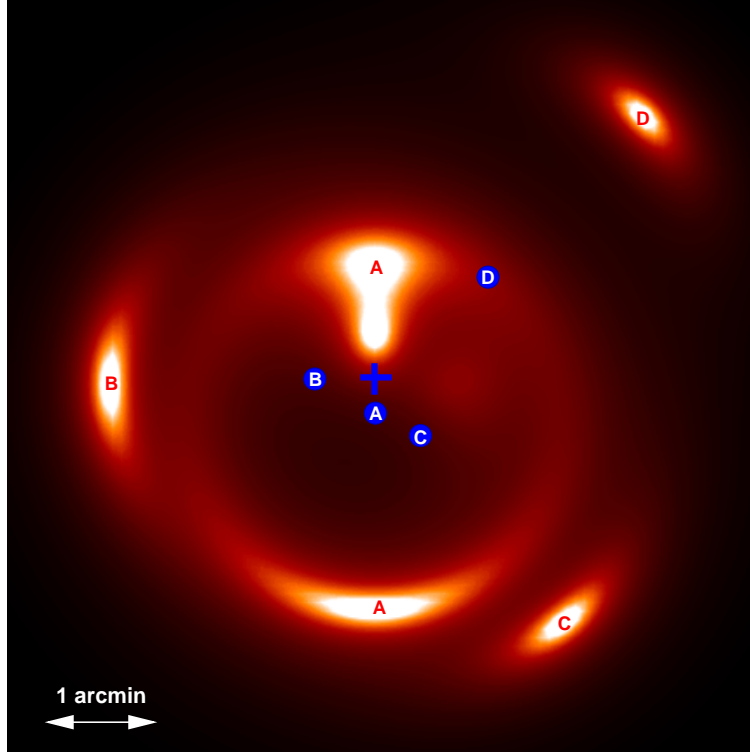


Figure 6: Lensing image of 4 sources at a redshift $z = 3.8$ by the rich cluster described in section 5. The blue bullets (A,B,C,D) show the positions of the sources as seen in the absence of the cluster. The blue cross marks the centre of the cluster. The source A placed on the lens caustic displays two images, one radially and the other tangentially elongated.

Aside from studying individual maps, we can also study the statistical effect that a population of such lenses would have on the power spectrum of the CMB fluctuations. This power spectrum is currently of immense theoretical and observational interest, since it is now a possibility that the spectrum may be measured, and so the values of cosmological constants may be found to unprecedented accuracy. Such determinations depend, however, on distinctive features in the power spectrum which may be affected by a population of large clusters. Previous work on this effect has been performed such as Seljak (1996), but we have now incorporated the effect of dynamically collapsing clusters in a proper general relativistic fashion. Figure 8 gives the result obtained with the cluster used in the current section.

We find that the distinctive peaks in the power spectrum are slightly smoothed out by such a population, and so this effect should be taken into account in future analyses.

6 Discussion

A CMB decrement of $\sim 380 \mu\text{Jy}$ has been observed by the Ryle-Telescope towards the pair of quasars PC1643+4631 A & B at red-shift ~ 3.8 separated by 100 arc-seconds (Jones et al. (1997)). Assuming a Sunyaev-Zel'dovich effect from a cluster of galaxies, this is indicative of a rich intervening cluster of total mass $\sim 10^{15} M_{\odot}$, although, no X-ray cluster has yet been observed in that direction. This suggests that the object is lying at a red-shift greater than one. The quasars' spectra analysis reveals that they might be originating from a unique source lensed by the distant cluster (Saunders et al. (1997)).

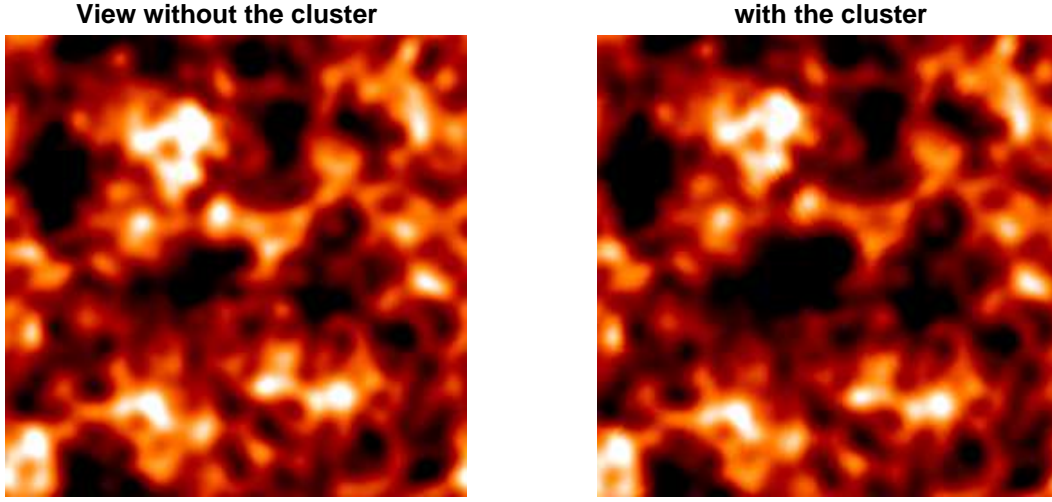


Figure 7: Effect of the collapsing cluster described in section 5 on the CMB fluctuations. The first map is a simulation of the CMB fluctuations due to inflation. The second one represents the same patch of the sky (4x4 degrees) but with a rich cluster of galaxies at its centre.

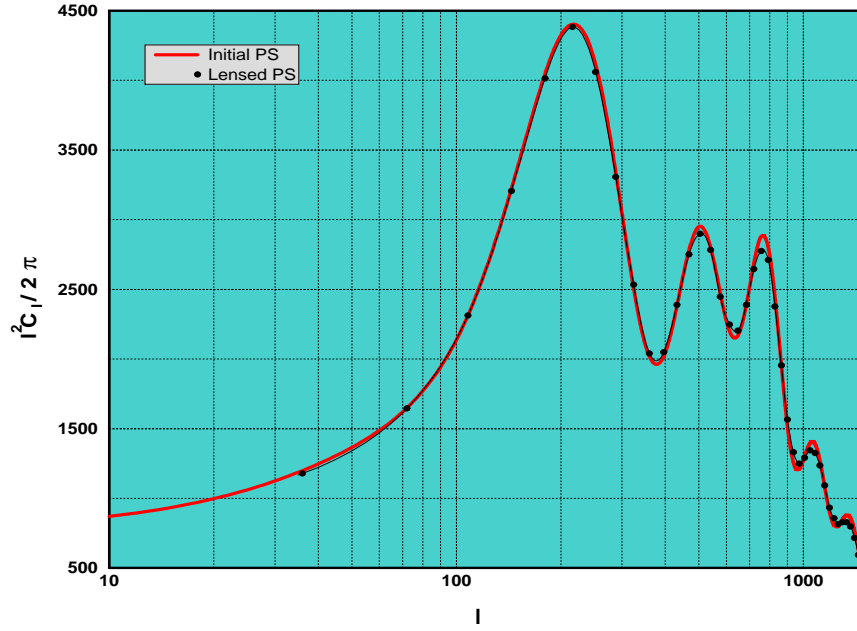


Figure 8: Effect on the CMB power spectrum of the rich cluster described in section 5. We assume one cluster per 10x10 degrees field. The red curve is the unperturbed power spectrum as the black dots represent the “lensed” spectrum.

Using our model, we are able to fit those observations and retrieve the S-Z flux decrement as well as the 100 arc-seconds separation. To do so, we have to consider the cluster described in section 5 with an electron temperature of 2×10^7 K, which corresponds to a rich but rather cool cluster. As we stated in section 5.1, the resulting Rees-Sciama temperature decrement is $\leq 180 \mu\text{K}$ which is a considerable fraction of the S-Z decrement, here $\sim 450 \mu\text{K}$. Fitting those data is of great importance since such a distant lensed-pair has previously never been observed. As the cluster lies at a rather large red-shift, we believe that a model such as ours, computing rigorously both the cluster and the universe evolutions, is needed.

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